Abstract

In this thesis we study the Krylov subspace methods applied to the solution of consistent singular systems. The methods in use today which are Galerkin methods or minimal residual seminorm methods, built help to the Hessenberg generalized process, call in some new results about the determination of a Krylov solution.

We show first, that in exact arithmetic or in finite precision, the determination of a Krylov solution depends on the initial vector used in the method and more precisely, on its decomposition on the characteristic subspaces associated to the matrix of the studied system.

Some precisions are also given about the breakdowns occuring in the GMRES method and on its restarted version.

Some new considerations are brought about the numerical treatment of some nonlinear partial differential equations depending on a parameter. In some cases, they call in singular systems whose matrix admits some Jordan blocks associated to the zero eigenvalue of maximal size 1, which is a sufficient condition to obtain a Krylov solution.
All these practical examples and their numerical treatment prove therefore that there exists alternatives to the breakdowns occuring in the algorithms.

Key words

Krylov subspace methods, generalized Hessenberg process, Hessenberg matrix, singular linear systems, breakdowns, partial differential equations, nonlinear systems, Newton method, continuation methods.